1.0

2.8   2.5

3.2

3.6   2.2

2.0

1.1

1.8

1.25   1.4   1.6

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TWO COMPUTER PROGRAMS FOR THE EVALUATION
OF THE ACOUSTIC PRESSURE AMPLITUDE AND PHASE
AT THE BOTTOM OF A WEDGE-SHAPED, FLUID
LAYER OVERLYING A FAST, FLUID HALF SPACE

by

A. B. Coppens, J. V. Sanders,
G. Ioannou, and M. Kawamura

December 1978

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Approved for public release, distribution unlimited

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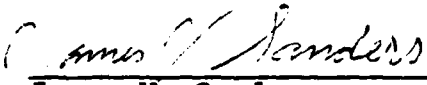
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The pressure amplitude and phase distribution along the inter-face between a tapered fluid layer and an underlying fast fluid bottom were investigated theoretically. Two computer models, both based on the method of images, were designed: WEDGE 1 assumes that the source is at a finite distance from the apex of the wedge, and WEDGE 2 assumes an infinite distance. The pressure amplitude along the interface rapidly falls off for

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(20) distances closer to the apex than the distance at which
adiabatic mode theory predicts cutoff for the lowest mode. This
distance is termed the "dump distance". The results of the two
programs agree whenever the source is several dump distances
from the apex. The predictions of these programs will be used
as input for a further program that will use the Green's
function approach to predict the acoustic pressure field in the
lower medium.

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1. INTRODUCTION

Sound radiated from a source located in a layer of fluid of
uniform depth overlying a second fluid with greater sound speed (a
"fast" bottom) can be trapped in the upper layer. This trapping
mechanism can be described by either ray or normal-mode theory: By
ray theory, if the angle of incidence of the ray on the bottom
exceeds the critical angle θ_c

$$\cos \theta_c = c_1/c_2$$

where θ is the grazing angle the ray makes with the bottom and
c_1 and c_2 are the speeds of sound in the top and bottom respec-
tively, the acoustic energy is totally reflected and no energy is
transmitted into the bottom. By normal mode theory, each normal
mode has associated with it a cutoff frequency

$$f_n = (c_1/2h)(n-1/2)\cosec \theta_c$$

where n is the mode number and h the thickness of the upper layer.
For frequencies above cutoff, the mode carries energy in the upper
layer; acoustic energy is trapped in the upper layer and propagates
to great distances. If both boundaries are smooth, the propagation
loss is only that associated with cylindrical spreading and
absorption losses.

If the upper layer is tapered so that it forms a wedge subtend-
ing an angle β, then as sound propagates towards the shallower
portion of the layer it will reach a depth at which transmission of
acoustic energy into the lower layer is possible. From the ray
point of view, a trapped ray will, on each encounter with the
sloping bottom, increase its angles of incidence, until, after a
sufficient number of encounters, the grazing angle will have

1

increased to a value greater than critical and energy enters the bottom. From the mode point of view, as the depth of the upper layer decreases, the cutoff frequency of a given mode increases until eventually the cutoff frequency exceeds that of the sound and energy enters the bottom. For a given frequency f, the thickness of the wedge at which the lowest mode attains cutoff is

$$h_1 = \lambda/(4 \sin \theta_c)$$

where $\lambda = c_1/f$ is the wavelength in the wedge. The distance from the apex measured along the interface at which the lowest mode attains cutoff, the "dump distance" is

$$X_o = h_1/\tan\beta = \lambda/(4 \sin \theta_c \tan\beta) \tag{1}$$

Obtaining an exact solution to the propagation of sound from the wedge into the bottom is difficult because the problem is not separable; refraction at the bottom interface results in components of the propagation vector at the bottom that depend on x. This precludes writing the boundary condition for the normal component of particle velocity at the interface in terms of one coordinate. As a result, the acoustic field cannot be considered as a product of functions of separate spatial coordinates.

Many of the usual approximate techniques also become inapplicable. For example, in the "adiabatic" assumption each normal mode in the wedge is assumed to retain its identity. This means that the problem can be treated using normal modes, whose propagation vectors will adjust slowly so that the propagation looks as if it were occurring in a layer of constant depth whose depth is the local depth. However, when cutoff is reached, the discrete set of normal

2

modes is replaced by the continuous set of untrapped modes, and the concept of modal identity loses its validity.

Our approach is to use the method of images to predict the amplitude and phase of the pressure at the bottom interface. The method of Green's functions is then used to obtain the beam pattern of the sound radiated into the bottom. The reflections of sound from the surface and bottom are replaced by images of the source in the same way as for a sound source in a fluid layer overlying an infinite fluid half-surface, a classical problem first solved by Pekeris. In the case of a wedge, the images lie on the circumference of a circle whose center is the apex of the wedge. The lowest images (closest to the source) correspond to rays of sound which make grazing reflections from the surfaces of the wedge. Higher images correspond to rays with greater angles of elevation and depression; these rays suffer more reflections from the surfaces of the wedge. Finally, images are encountered for which the rays exceed the critical angle at the bottom. Still higher images correspond to more reflections from the bottom (with reflection coefficients less than one in magnitude) so the effective strength of these higher images will be progressively reduced. (This avoids certain problems involved with diffraction from the apex of the wedge.) Phase coherent summation of the contributions from the source and images yields the desired pressure and phase at the bottom of the wedge.

Once the pressure amplitude and phase distribution on the interface are known, the interface can be treated as an apparent acoustical source. Application of straightforward Green's-function

techniques then results in an integral expression for the sound field in the bottom,

$$p(\vec{r}, t) = \frac{e^{j\omega t}}{4\pi} \int_{-\infty}^{\infty} P(x') e^{j[k_2 x' + \Omega(x')]} \frac{\partial}{\partial \zeta'} \left[j\pi H_0^{(2)}(k_2 |\vec{r} - \vec{r}'|) \right]_{\zeta' = 0} dx'$$

$$|\vec{r} - \vec{r}'| = \sqrt{(x - x')^2 + (\zeta - \zeta')^2}$$

(1)

Under the assumption that the field in the bottom is studied for $|\vec{r} - \vec{r}'| \gg \lambda$, these equations can be well approximated by

$$p(\vec{r}, t) = \frac{e^{j(\omega t + \frac{\pi}{4})}}{4} \sqrt{\frac{2 k_2 X_0}{\pi}} \, \eta \int_0^{\infty} \frac{P(\xi') \exp\left\{ j \left[k_2 X_0 \xi' - \sqrt{(\xi - \xi')^2 + \eta^2} \right] + \Omega(\xi') \right\}}{\left[(\xi - \xi')^2 + \eta^2 \right]^{3/4}} \, d\xi'$$

$$\xi = x/X_0, \quad \xi' = x'/X_0, \quad \eta = \zeta/X_0$$

(2)

where

$$k_2 X_0 = \frac{\pi/2}{\sin \theta_c \, \tan \beta}$$

and in the above integrals

$$\underline{P}(\xi') e^{j\omega t} = P(\xi') e^{j[\omega t + k_2 X_0 \xi' + \Omega(\xi')]}, \quad \xi > 0$$

(3)

is the image-calculated pressure at the interface and we must have $P(\xi') = 0$ for $\xi' < 0$ to satisfy the boundary condition to the right of the apex of the wedge.

The first stage of this investigation, the subject of this report, is the prediction of the pressure and phase distribution along the interface.

2. THEORY

a. Finite distance from source to receiver

Figure 1 shows a wedge with a pressure-release surface and a fast bottom. The origin of coordinates is at the apex with the x axis along the bottom, positioned at the left, and the y axis positioned downward. A source is a distance x from the axis and a depth H below the sea surface. The line connecting the source to the apex makes an angle φ_d with the sea surface and an angle

$$\varphi_o = \beta - \varphi_d \tag{4}$$

with the bottom. The distance between a receiving point on the bottom at distance x from the nth image is

$$r_n = \sqrt{X^2 + x^2 - 2x X \cos \varphi_n} \tag{5}$$

where φ_n is the angle between the line joining the image to the apex and the sea bottom. (See Fig. 1.) Each φ_n is given by

$$\varphi_n = 2 \, INT\left[\frac{n+1}{2}\right]\beta + (-1)^n \varphi_o \tag{6}$$

where INT[ ] denotes the largest integer which is equal to, or smaller than the argument. The number of times an image interacts with the bottom is equal to the number of times the line from the image to the receiving point intersects the images of the bottom. The grazing angle on the bottom for the mth bounce (counted away from the receiving point) for the nth image is the angle between the line joining the nth image to the receiver and the mth image of the bottom,
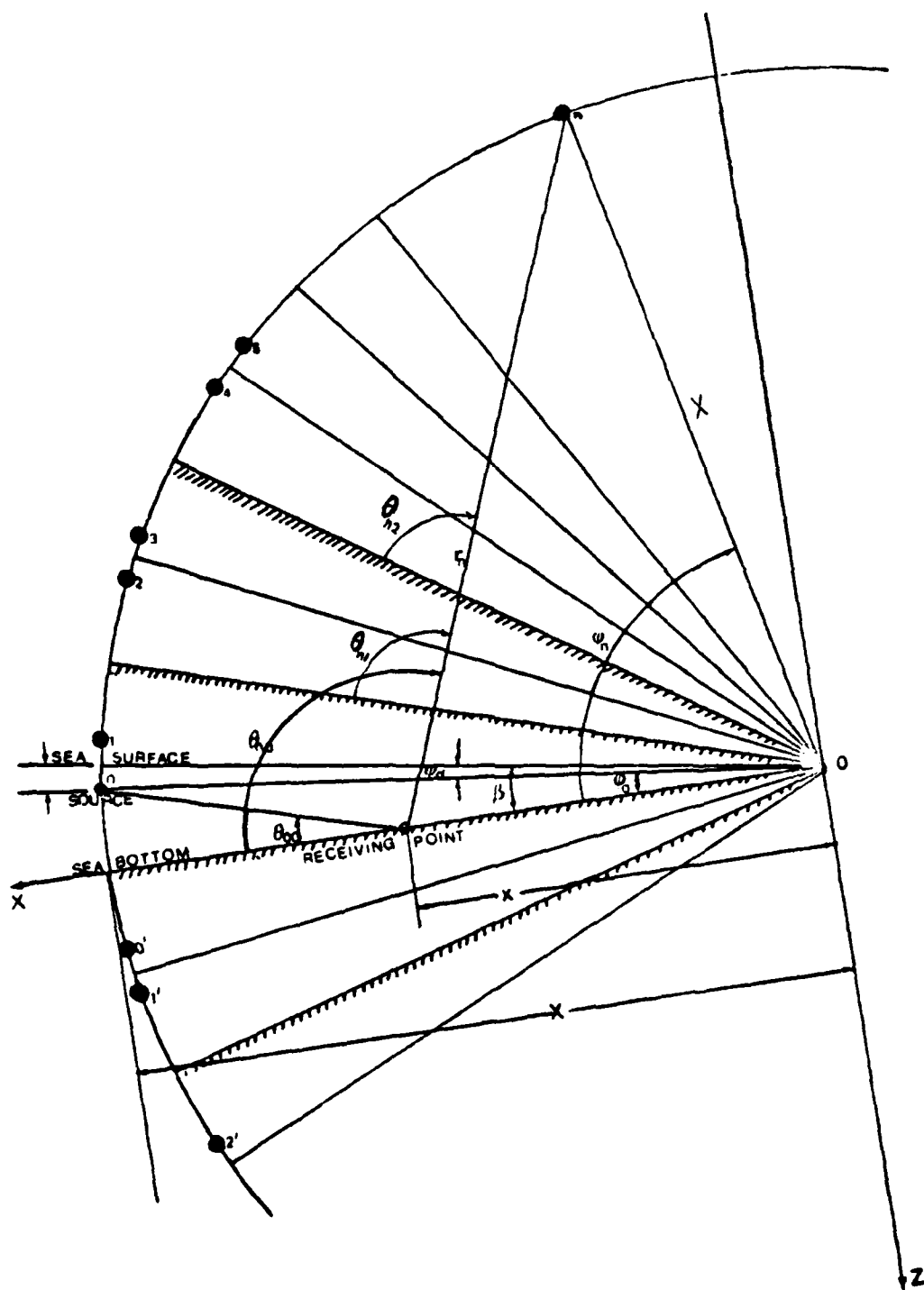
Fig. 1 The Geometry of the image solution

$$\theta_{nm} = \theta_{no} - 2m\beta \tag{7}$$

where θ_{no} is the grazing angle at the receiving point.

If the distance from the source to the point on the bottom at which the initial bounce takes place is large compared to a wavelength in the wedge, the reflection coefficient for the nth image at the mth bounce, can be approximated by the plane-wave reflection coefficient,

$$R_{nm} \doteq \frac{\dfrac{\rho_1 c_1}{\rho_2 c_2} - \psi_{nm}}{\dfrac{\rho_1 c_1}{\rho_2 c_2} + \psi_{nm}} \tag{8}$$

where c_1, c_2, ρ_1, ρ_2 are the sound speeds and densities of the two media, and

$$\psi_{nm} = \frac{\sqrt{1 - \left(\dfrac{c_2}{c_1}\right)^2 \cos^2 \theta_{nm}}}{\sin \theta_{nm}} \quad , \quad \theta_{nm} \geq \theta_c = \cos^{-1}\left(\frac{c_1}{c_2}\right)$$

$$\psi_{nm} = -\frac{j\sqrt{\left(\dfrac{c_2}{c_1}\right)^2 \cos^2 \theta_{nm} - 1}}{\sin \theta_{nm}} \quad , \quad \theta_{nm} < \theta_c \tag{9}$$

For a source of unit pressure at 1m from its center, the complex pressure $\underline{p}(x,t) = \underline{P}(x)e^{j\omega t}$ infinitesimally above the bottom has complex amplitude

$$\underline{P}(r) = \frac{1}{r_0} e^{-jkr_0}$$
$$+ \sum_{n=0}^{N} \left[\frac{1}{r_n} e^{-jkr_n} (-1)^{INT\left[\frac{n+1}{2}\right]} \prod_{m=1}^{M_n} \underline{R}_{nm} \right]$$
$$+ \sum_{n=0}^{N} \left[\frac{1}{r_n} e^{-jkr_n} (-1)^{INT\left[\frac{n+1}{2}\right]} \prod_{m=0}^{M_n} \underline{R}_{nm} \right] \tag{10}$$

where the first term is the contribution from the direct path, the second term is the contribution from the images above the wedge and the last term denotes the contribution of the images below the wedge; k is the wave number in the wedge;

$$M_n = INT\left[\frac{\varphi_n}{\beta}\right], \qquad N = INT\left[\frac{\pi}{\beta}\right] \tag{11}$$

and the $(-1)^{INT[\ ]}$ accounts for the phase shift on each interaction with the pressure release surface. The second and third terms differ only in that there is a 0th bounce associated with the images above the wedge.

The complex pressure amplitude along the bottom can be written as a simple sum

$$\underline{P}(r) = \sum_{n=1}^{N} \frac{1}{r_n} e^{-jkr_n} (-1)^{INT\left[\frac{n+1}{2}\right]} \left(1 + \frac{1}{\underline{R}_{n0}} \right) \prod_{m=0}^{M_n} \underline{R}_{nm} \tag{12}$$

Computer program (WEDGE 1) calculates the pressure-amplitude and phase-distribution along the bottom resulting from a source at a finite distance from the receiver.

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A refinement which includes the effect of a directional source is presented in M. Kawamura and I. Ioannou, "Pressure on the Interface Between a Converging Fluid Wedge and a Fast Fluid Bottom", M.S. Thesis, NPS, (Dec. 1978) and named WEDGE O. In both of these programs the complex pressure amplitude $\underline{P}(x)$ was "normalized" by multiplying the right hand side of Eq(9) by r_0. This has the effect of assuring that the source generates a pressure amplitude at one meter which is proportional to its distance from the apex. The advantage is that the effects on $r_0\underline{P}(x)$ of changing r_0 are easily studied without having to correct for the $1/r_0$ diminuation in $\underline{P}(x)$.

b. Infinite distance from source to receiver.

In the case of a distant source, inspection of Fig. 2 yields the approximations

$$\theta_{no} \doteq \varphi_n$$

$$\theta_{nm} \doteq \varphi_n - 2m\beta$$

(13)

Now, since

$$\underline{R}_{no} = \underline{R}(\theta_{no}) \doteq \underline{R}(\varphi_n)$$

(14)

and the expressions for the reflection coefficients are

$$\prod_{m=0}^{M} \underline{R}_{om} = \underline{R}_o = \underline{R}(\varphi_o)$$

$$\prod_{m=0}^{n} \underline{R}_{1m} = \underline{R}_1 = \underline{R}(\varphi_1)$$

$$\prod_{m=0}^{n} \underline{R}_{2m} = \underline{R}_o \underline{R}_1 = \underline{R}(\varphi_o)\underline{R}(\varphi_1)$$

(15)

$$\vdots$$

$$\prod_{m=0}^{M} \underline{R}_{nm} = \prod_{m=0}^{n} \underline{R}_{n-2,m} \underline{R}_n = \prod_{m=0}^{M} \underline{R}(\varphi_{n-2})\underline{R}(\varphi_n)$$

9
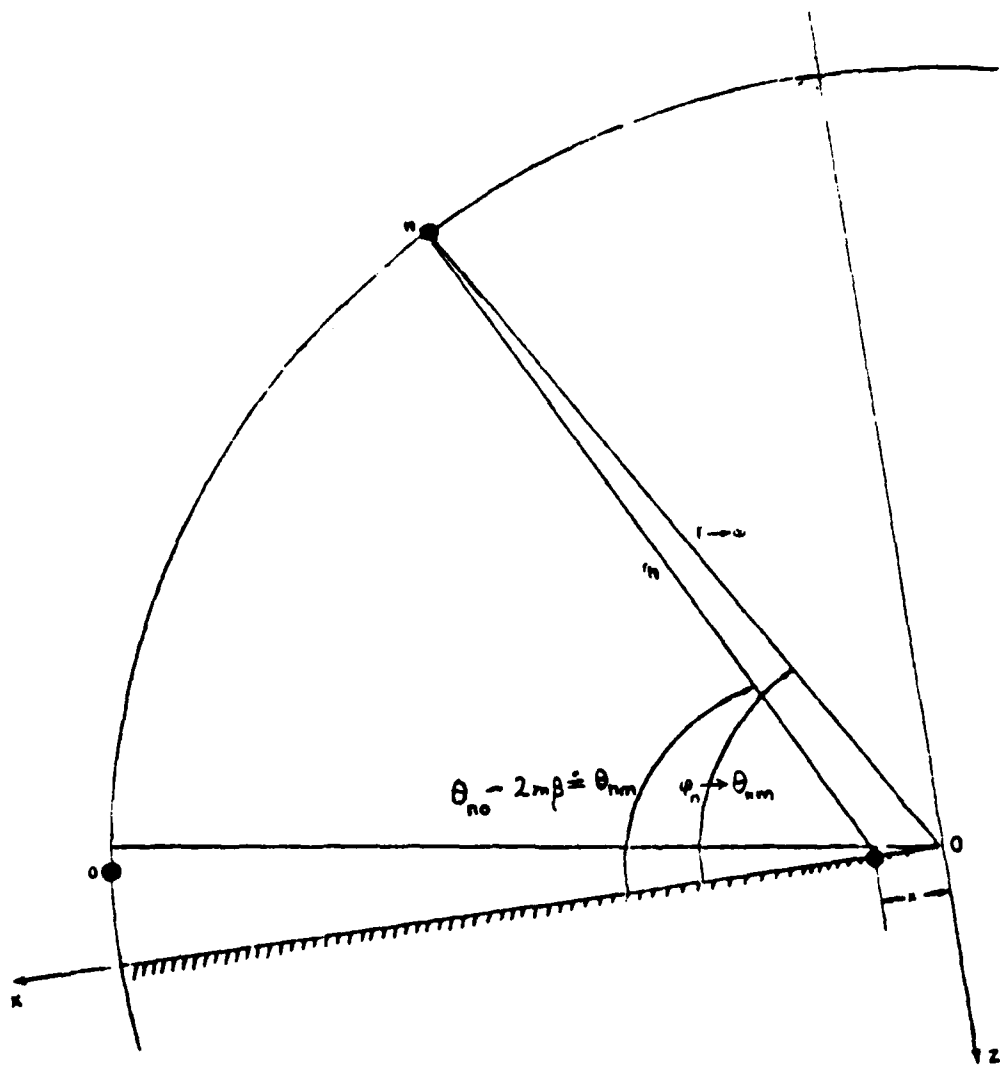
Fig. 2 Geometry of the distant source approximation

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The right hand sides of Eq. 15 depend only on n, so we can define

$$\underline{\Gamma}(n) \doteq \begin{cases} \underline{R}_0 & n = 0 \\ \underline{R}_1 & n = 1 \\ \underline{\Gamma}(n-2)\, \underline{R}_n & n = 2, 3, \cdots \end{cases} \tag{16}$$

Also, the geometry of Fig. 2 gives

$$r_n \doteq r - x \cos \varphi_n \tag{17}$$

Substitution of Eqs. (15), (16) and (17) into Eq. (12) yields

$$\underline{P}(x) \doteq \frac{1}{r}\, e^{-j k r} \sum_{n=0}^{N} e^{j k x \cos \varphi_n} \, (-1)^{INT\left[\frac{n+1}{2}\right]} \left(1 + \frac{1}{R_n}\right) \underline{\Gamma}(n) \tag{18}$$

Computer program (WEDGE 2) evaluates Eq. (18) and can be used to
calculate the pressure amplitude and phase distribution along the
bottom resulting from a source infinitely distant from the
receiver.

As in WEDGE 1, what was actually computed was the "normalized"
complex pressure amplitude $r\underline{P}(x)$.

3. TESTING.

   a. Verification of WEDGE 1

To verify that WEDGE 1 was performing the calculations correct-
ly, several runs were made for bottoms with high impedances to
approximate a rigid boundary. For these runs the wedge angle was
chosen to produce a limited number of images so that calculation of
the pressure was simple enough to be performed on a hand calculator.

Runs were made for $90°$ and $45°$ wedge angles. In all cases the
agreement between hand calculation and WEDGE 1 output was considered
to be very good. An example is given in Figs. 3 and 4 for a $45°$
wedge $c_1/c_2 = 0.2$ and $\rho_1/\rho_2 = 0.05$. In these and the following
figures the pressure amplitude is normalized so that its greatest
value is unity. (This normalization was not done in WEDGE 0.)

   b. Verification of WEDGE 2

Having affirmed that WEDGE 1 was producing valid output for the
pressure amplitude and phase distribution at the interface between
the wedge and the bottom, comparisons between WEDGE 1 and WEDGE 2
were made. Because WEDGE 2 is valid only for distant sources,
comparisons were performed for source distances greater than 100
times the dump distance X_0. One example of this comparison is
given in Fig. 5. WEDGE 2 results are plotted only where, by visual
inspection, they do not exactly overlap the results of WEDGE 1.

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Rigid boundary
slope angle = 45° c_1 = 1000.0 m/s
source depth = 50.0 m c_2 = 5000.0 m/s
source dist. = 100.0 m ρ_1 = 1000 kg/m³
frequency = 50.0 kHz ρ_2 = 20,000 kg/m³
lowest mode cut off depth = 5.103 m
lowest mode cut off distance = 7.217 m
                    from the apex

. WEDGE 1

+ Direct Computation

Normalized distance (x/x_o)

Pressure Amplitude (Normalized)
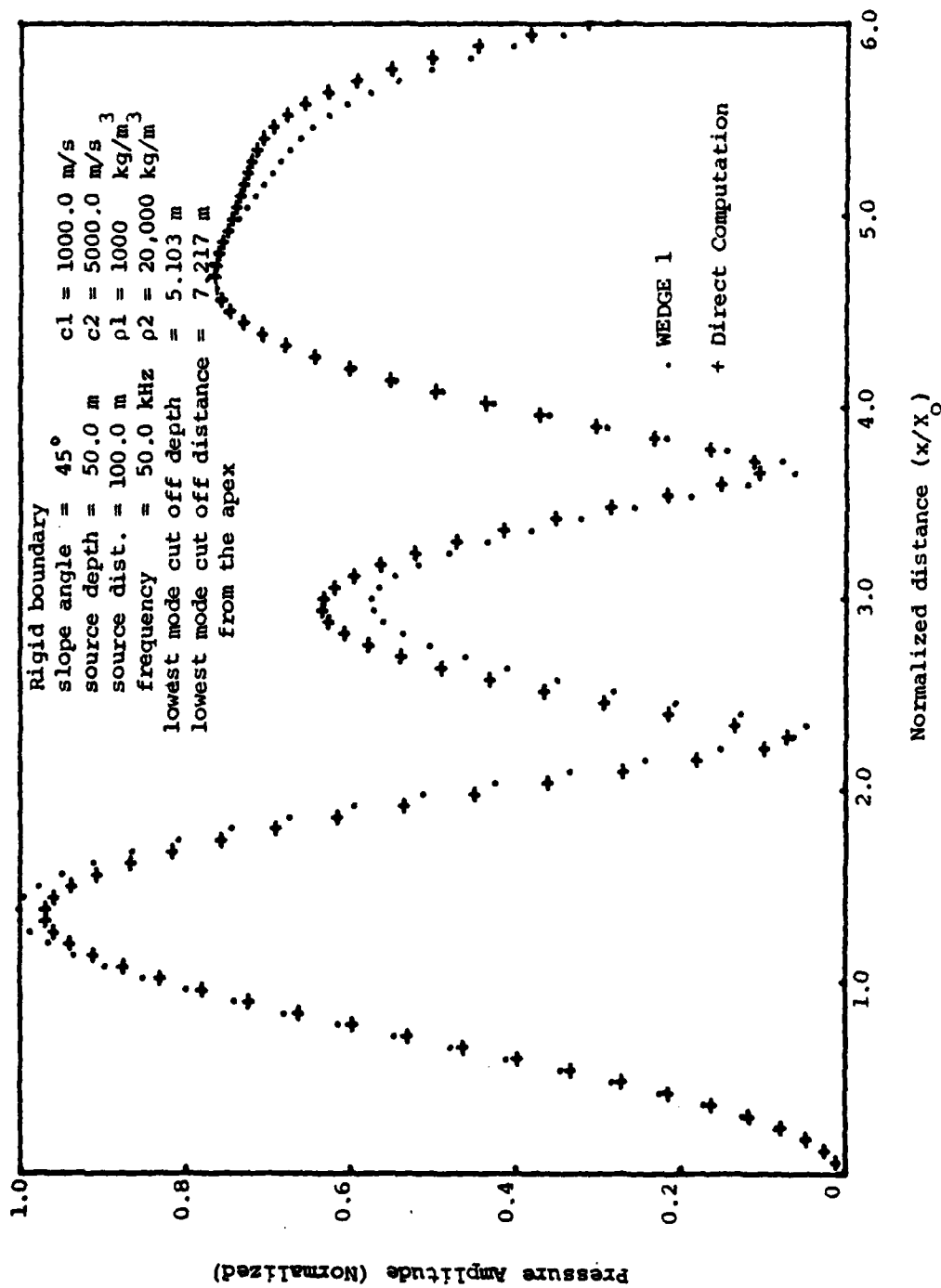
Fig. 3 Comparison of pressure amplitudes

13
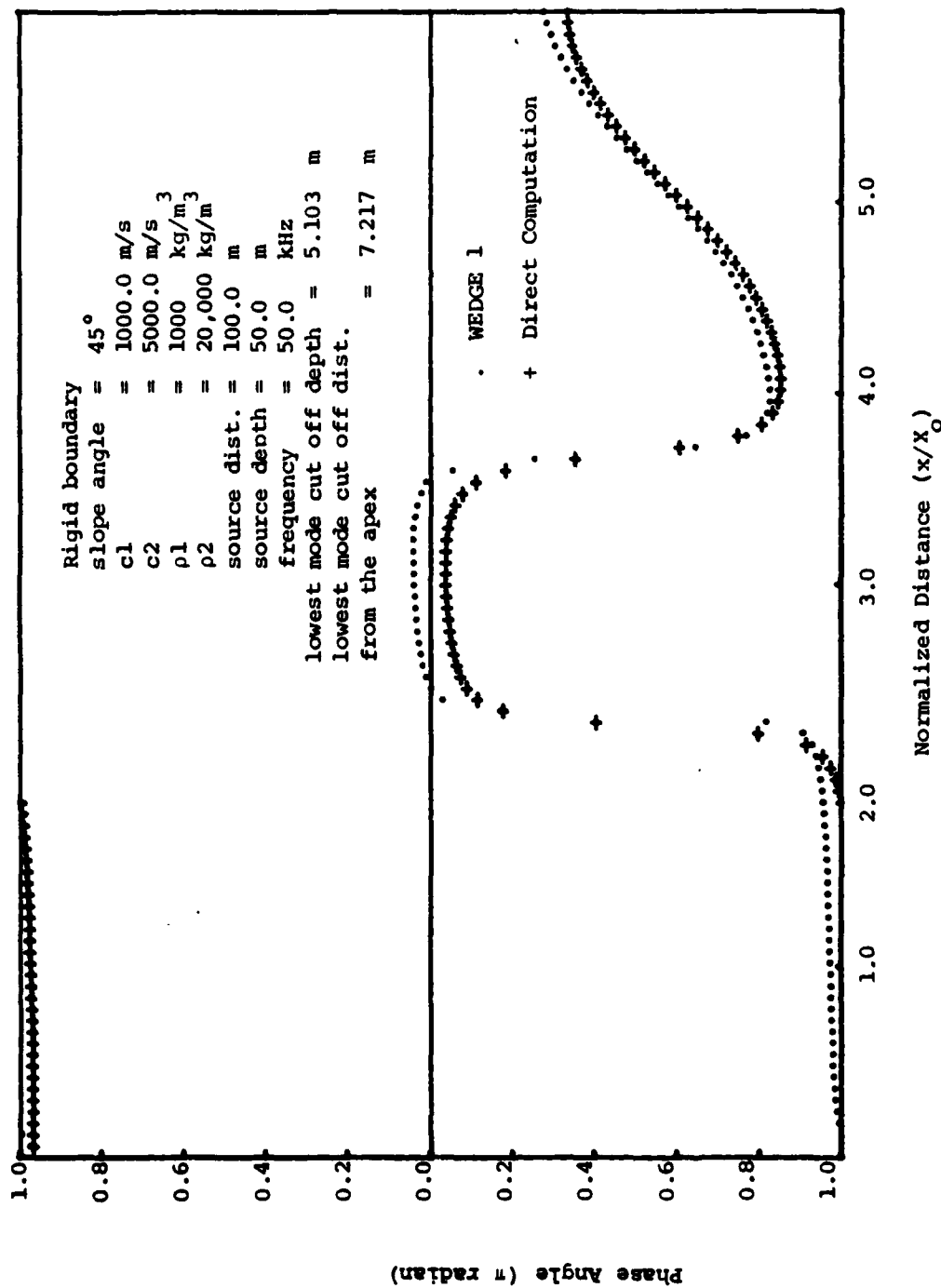
Fig. 4 Comparison of phase angles

Rigid boundary
slope angle   =   45°
c_1 =   1000.0 m/s
c_2 =   5000.0 m/s
ρ_1 =   1000   kg/m^3
ρ_2 =   20,000 kg/m^3
source dist.   =   100.0   m
source depth   =   50.0   m
frequency   =   50.0   kHz
lowest mode cut off depth =   5.103   m
lowest mode cut off dist.
from the apex   =   7.217   m

·   WEDGE 1

+   Direct Computation

Normalized Distance (x/x_o)

Phase Angle (π radian)

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wedge angle   =   3°
c1   =   1500.0 m/s
c2   =   1730.0 m/s
ρ1   =   1025   kg/m³
ρ2   =   2070   kg/m³
source depth   =   24.24   km }WEDGE 1
source dist.   =   926   km }
source elevation angle = 1.5° WEDGE 2
frequency   =   10 kHz
lowest mode cut off depth   =   75.27 m
lowest mode cut off distance = 1431.62 m

+ WEDGE 1

● WEDGE 2

Pressure Amplitude (Normalized)

Normalized Distance from apex (x/x$_o$)

Fig. 5 Comparison of pressure amplitudes

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For the WEDGE 1 calculations the source was located 647 dump distances from the apex and at mid-depth. For WEDGE 2, the source, defined to be at infinity, was positioned at mid-depth so that the same normal modes were excited in each case.

In all cases investigated, the amplitudes of the pressure on the bottom calculated by the two programs were in excellent agreement.

Phase results are not displayed because the phases fluctuated so rapidly with respect to the increment in distance that no discernable pattern could be seen. However, detailed comparisons of the numerical outputs of WEDGE 1 and WEDGE 2 confirmed that the two programs were also in agreement in their calculations of the phases of the pressures at the interface.

4. DISCUSSION OF RESULTS

Figures 5 and 6 show the pressure amplitude (normalized to give a peak value of unity) as a function of distance from the apex (normalized by dividing by the dump distance). Both figures are for the same wedge geometry and wedge and bottom properties; they differ only in the location of the source. In Fig. 5 the WEDGE 1, source is at 647 dump distances and at mid-depth. In Fig. 6 WEDGE 1, the source is at 64.7 dump distances and at a depth of 1000 m (one-fifth of the depth at that distance). In both figures the WEDGE 2 calculation was carried out with the source position to give the same source angle as the corresponding WEDGE 1 calculations.

16

+ WEDGE 1

⊙ WEDGE 2

Pressure Amplitude (Normalized)
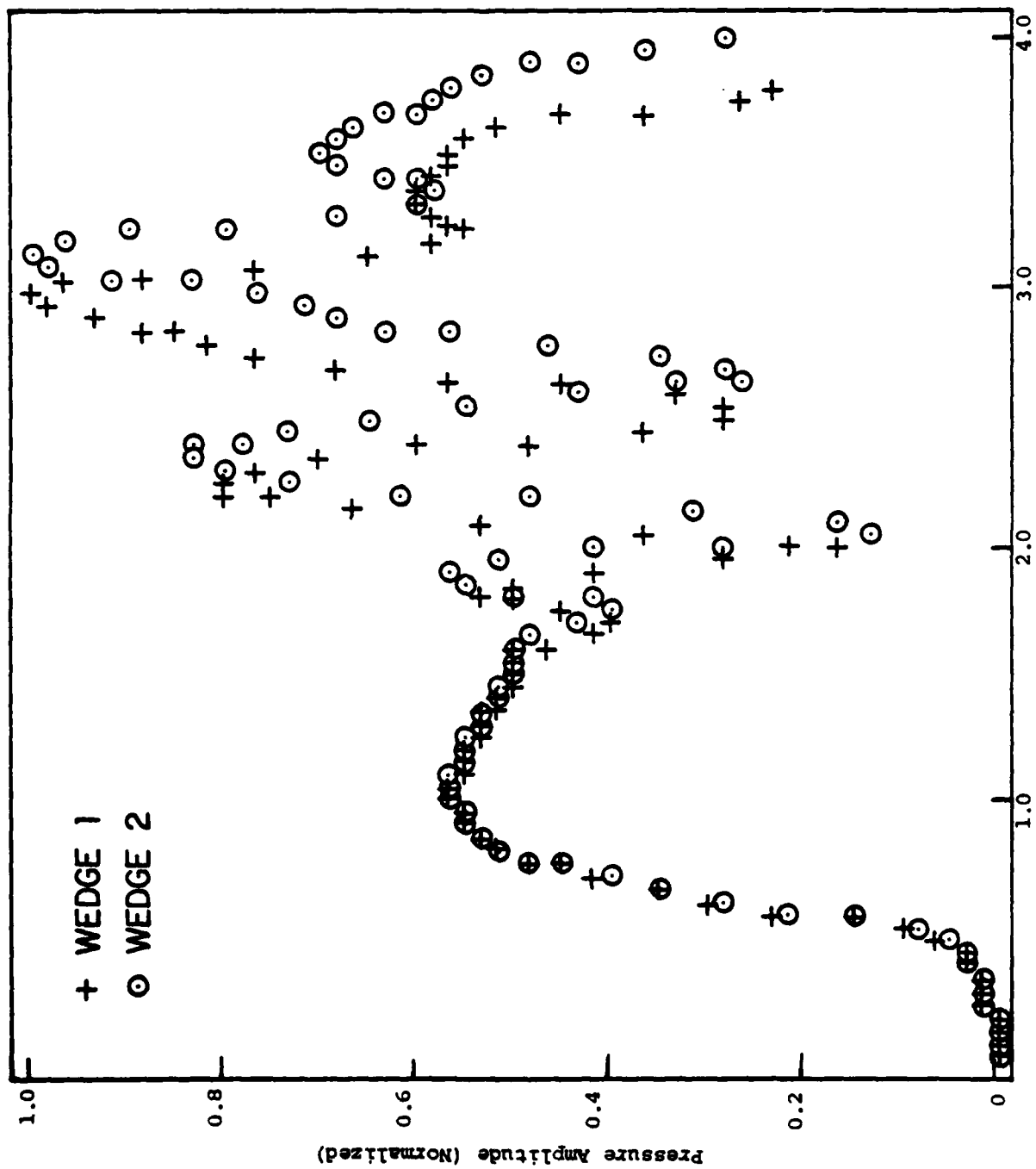
Fig. 6 Comparison of pressure amplitudes

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In all cases the pressure amplitude on the bottom increases smoothly from zero at the apex to a local maximum at the dump distance. The pressure amplitude then decreases at distances further from the apex until modal interference becomes important at about two dump distances when the position of the source is such that the second mode is strongly excited (Fig. 6) and at about three dump distances when the second mode is weakly excited (Fig. 5).

Comparison of the results for WEDGE 1 and WEDGE 2 in Fig. 6 shows that there is little difference between the pressure distribution produced by a source at 65 dump distances and one infinitely distant; the positions and magnitudes of the the interferences peaks are slightly different, but the results out to about two dump distances are indistinguishable in the two cases.

5. CONCLUSIONS

Two programs have been generated to calculate the pressure amplitude and the phase distribution on the interface between a fluid wedge and an underlying semi-infinite fluid medium. The first program, WEDGE 1, is the more general in that it allows the source to be positioned anywhere within the wedge. The second WEDGE 2 is more restricted in that the source is assumed to be infinitely distant from the apex of the wedge. In practice this means that for reasonably good predictions of the pressure and phase at the inter-face out to a couple of times the dump distance X_O, the source should lie more than about $100\,X_O$ away from the apex. In running comparable cases, the difference in computation time between the two programs amounts to a factor of five, with WEDGE 2 being the faster of the two.

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Equations for the computer program (WEDGE 1)

$\pi = 4 \tan(1)$ constant

$K = 2 \pi f / C$ wave number

$r = \rho C$ acoustic impedance

$\varphi_0 = \beta - \sin(H/X)$ angle between source and bottom

$\varphi_n = 2 \, \text{INT}\,[\,(n+1)/2\,]\,\beta + (-1)^n$ *   angle between source and n'th image

$\theta_{no} = \tan^{-1}[\,\sin\varphi_n / (\cos\varphi_n - x/X)\,]$ incident angle to the bottom (last reflection)

$D_n = \sqrt{X^2 + x^2 - 2\,x\,X\cos\varphi_n}\,/X$ normalized distance between image and receiver

$M = \text{INT}\,(\varphi_n / 2\beta)$ bottom reflections

$\theta_{nm} = \theta_{no} - 2\,m\,\beta$ incident angle to the bottom (intermediate reflections)

$\psi_1 = \sqrt{1 - (C_2/C_1)^2 \cos^2\theta_{nm}}\,/\,\sin\theta_{nm}$ parameter for a reflection coefficient ($\theta_{nm} \geq \theta_c$)

$\psi_2 = -\,i\,\sqrt{(C_2/C_1)^2 \cos^2\theta_{nm} - 1}\,/\,\sin\theta_{nm}$ parameter for a reflection coefficient ($\theta_{nm} < \theta_c$)

$\psi = \psi_1 \ \text{or} \ \psi_2$

$R_{nm} = (r_2/r_1 - \psi_{nm}) / (r_2/r_1 + \psi_{nm})$ reflection coefficent

$P = \sum_{n=0}^{n}\,[\,1/D_n \cdot e^{-ikxD_n}(-1)^{\text{INT}\left(\frac{n+L}{2}\right)}\,(1+1/R_{no})\prod_{m=0}^{M}(R_{nm})\,]$ **   pressure at point (x,0)

$P = |P|$ pressure amplitude

$\underline{/P} = \tan^{-1}[\,\text{Im}(P)/\text{Re}(P)\,]$ pressure phase   P

* INT( )   max. integer of an argument

      $2\,\text{INT}\,(n+1)/2 = n + 1/2\,[\,1 - (-1)^n\,]$

** $\text{INT}\,(n+1)/2 = 1/4\,[\,2n + 1 - (-1)^n\,]$

FLOW CHART FOR SUBROUTINE 'PRES'
(WEDGE 1)
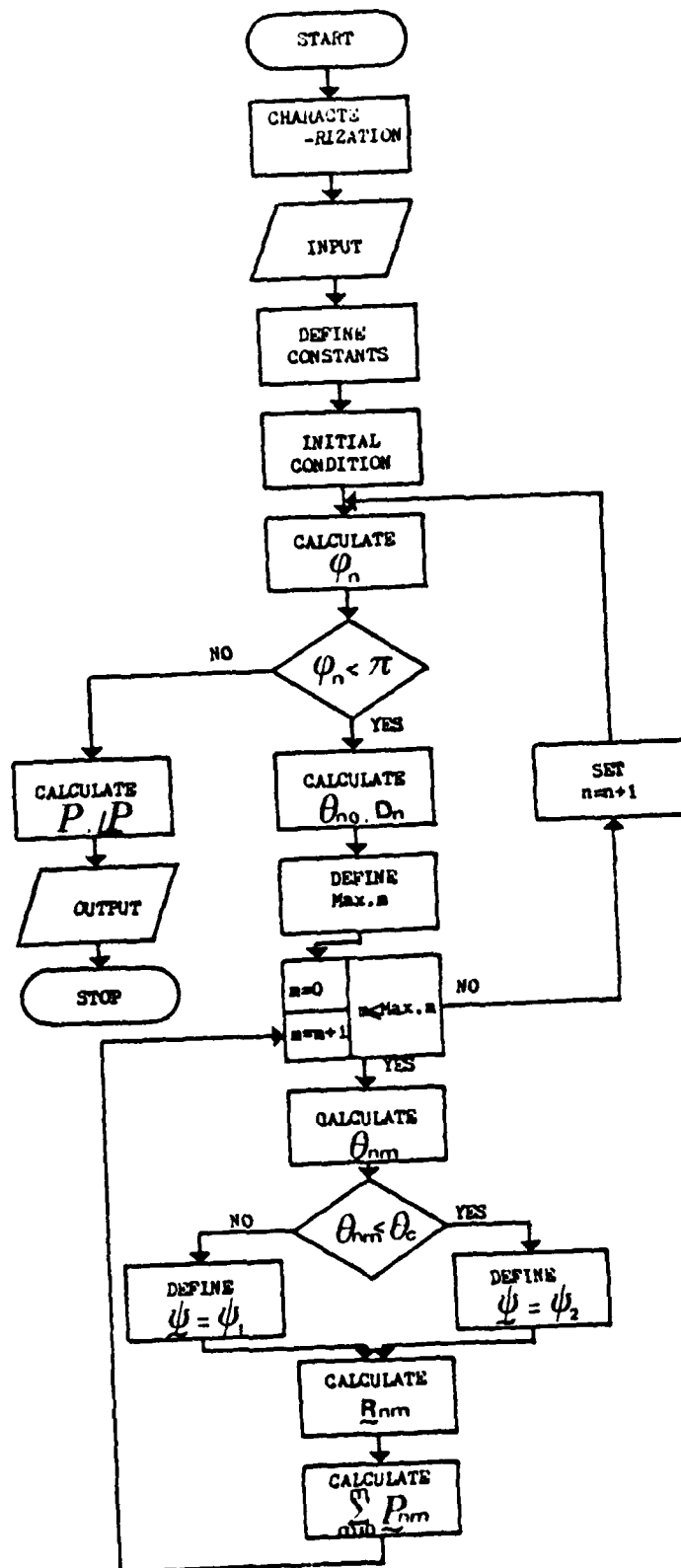
```
C      WEDGE 1
C *** MAIN PROGRAM ***
C
C     **********************************************************************
C     * THIS PROGRAM EMPLOYS THE METHOD OF IMAGES TO OBTAIN THE PRESSURE   *
C     *AND PHASE DISTRIBUTION ALONG THE BOTTOM OF A WEDGE-SHAPED FLUID      *
C     *LAYER,OVERLYING A FAST BOTTOM.                                      *
C     *                                                                    *
C     *  ASSUMPTIONS                                                       *
C     * (1) PLANE WAVE REFLECTION COEFICIENTS.                             *
C     * (2) SOURCE IS POINT SOURCE.                                        *
C     **********************************************************************
C
C     *  NOTATION
C        I/O   OUT=OUTPUT
C              IN =INPUT
C        CHARACTER
C              V  =VARIABLE
C              P  =PARAMETER
C              C  =CONSTANT
C
C              R  =REAL NUMBER
C              I  =INTEGER
C              Z  =COMPLEX NUMBER
C
C     ------------------------------------------------------------------
C     SYMBOL          MEANING                                  I/O CHARACTER
C     ------------------------------------------------------------------
C     PA       PRESSURE AMPLITUDE ON THE BOTTOM               OUT   V,R
C     PH       PRESSURE PHASE ON THE BOTTOM                   OUT   V,R
C     C1       SOUND SPEED IN MEDIUM 1 (WEDGE)                IN    C,R
C     C2       SOUND SPEED IN MEDIUM 2 (BOTTOM)               IN    C,R
C     RO1      DENSITY OF MEDIUM 1                            IN    C,R
C     RO2      DENSITY OF MEDIUM 2                            IN    C,R
C     BETA     SLOPE ANGLE OF BOTTOM                          IN    C,R
C     X        SOURCE DISTANCE FROM THE APEX IN METERS        IN    C,R
C     XX       DISTANCE FROM THE APEX IN METER               OUT   V,R
C                AT WHICH PA AND PH ARE CALCULATED
C     F        DRIVING FREQUENCY                              IN    C,R
C     H        SOURCE DEPTH IN METERS                         IN    C,R
C     ANGLC    CRITICAL ANGLE                                       C,R
C     HO       LOWEST POSSIBLE MODE CUT OFF DEPTH IN METERS         C,R
C     XO       LOWEST POSSIBLE MODE CUT OFF DISTANCE FROM THE       C,R
C                APEX IN METERS
C     X1       DISTANCE FROM APEX AT WHICH CALCULATION STARTS IN    C,R
C     X2       DISTANCE FROM APEX AT WHICH CALCULATION STOPS  IN    C,R
C     DX       RANGE OF CALCULATION IN METERS                       C,R
C     N        NUMBER OF POINTS FOR WHICH PA AND PH ARE       IN    C,I
C                CALCULATED
C     PAI      3.1415.......                                        C,R
C     WN       WAVE NUMBER                                          C,R
C     C21      SOUND SPEED RATIO =C2/C1                             C,R
C     RC21     ACOUSTIC IMPEDANCE RATIO                             C,R
C     ANGLO    ANGLE FORMED BY SOURCE, APEX AND BOTTOM              C,R
C     DISTX    TEMPORARY FOR XX(I)/X                                P,R
C     PAMP     TEMPORARY FOR PA                                     P,R
C     PHAS     TEMPORARY FOR PH                                     P,R
C     ------------------------------------------------------------------
C
C
C     **********************************************************************
```

```
C
C      ((PROGRAM))
C
C DEFINE CHARACTERS OF VARIABLES,PARAMETERS AND CONSTANTS
C
      IMPLICIT REAL*8(A-H,O-Z),INTEGER(I-N)
      DIMENSION PA(101),PH(101),XX(101)
      COMMON C21,RC21,BETA,WN,PAI,ANGLO,X,H,F
C
C READ IN DATA
C
      READ(5,100) C1,C2,RO1,RO2,BETA
      READ(5,500) X1,X2,X,H,F
      READ(5,600) N
C
C CALCULATE AND WRITE CONSTANTS
C
      WRITE (6,700) C1,C2,RO1,RO2,X1,X2,X,H,BETA,F
      ANGLC=DARCOS(C1/C2)
      HO=C1/(4.0D0*F*DSIN(ANGLC))
      XO=HO/DSIN(BETA)
      WRITE(6,400) HO,XO
C
C CALCULATE RANGE DX
C
      DX=X2-X1
C
C CALCULATE CONSTANTS FOR SUBROUTINE
C
      PAI=4.0D0*DATAN(1.0D0)
      WN=2.0D0*PAI*F/C1
      C21=C2/C1
      RC21=C2*RO2/(C1*RO1)
      ANGLO=BETA-DARSIN(H/X)
      WRITE (6,200)
C
C CALCULATE PRESSURES WITH RESPECT TO DISTANCE XX
C  WHERE XX IS CHANGED BY INCRIMENT
C
      L=N+1
      DO 10I=1,L
      XX(I)=X1+DFLOAT(I-1)*DX/DFLOAT(N)
      DISTX=XX(I)/X
      CALL PRES (DISTX,PAMP,PHAS)
      PA(I)=PAMP
      PH(I)=PHAS
      WRITE (6,300) XX(I),PA(I),PH(I)
   10 CONTINUE
      WRITE (6,800)
C
C PLOT THE PRESSURE AMPLITUDE VS. DISTANCE XX
C  AND PRESSURE PHASE VS. DISTANCE XX
C
      WRITE (6,700) C1,C2,RO1,RO2,X1,X2,X,H,BETA,F
      CALL DPLTP (XX,PA,N,0)
      WRITE (6,800)
      WRITE (6,700) C1,C2,RO1,RO2,X1,X2,X,H,BETA,F
      CALL DPLTP (XX,PH,N,0)
C
```

```fortran
C I/O FORMATS
C
  100 FORMAT (5F15.0)
  200 FORMAT (//22X,'DISTANCE',17X,'PRESSURE AMP.',12X,'PHASE ANGLE'//)
  300 FORMAT (  20X,D15.7,10X,D15.7,10X,D15.7)
  400 FORMAT (//20X,'LOWEST POSSIBLE MODE CUT OFF DEPTH',9X,D15.7
     C          /20X,'DISTANCE FROM APEX        ',20X,D15.7//)
  500 FORMAT (5F15.0)
  600 FORMAT (I3)
  700 FORMAT (//25X,'INPUT DATA',15X,'WEDGE1'//
     C          25X,'C1  =',D15.7,5X,'C2  =',D15.7/
     C          25X,'RO1 =',D15.7,5X,'RO2 =',D15.7/
     C          25X,'X1  =',D15.7,5X,'X2  =',D15.7/
     C          25X,'X   =',D15.7,5X,'H   =',D15.7/
     C          25X,'BETA=',D15.7,5X,'F   =',D15.7/)
  800 FORMAT (1H1)
      STOP
      END
```

```
C *** SUB PROGRAM ***
C
C   **********************************************************************
C   * SUBROUTINE 'PRES' CALCULATES THE PRESSURE AMPLITUDE AND PHASE AT A*
C   *POINT ON THE BOTTOM BY USING METHOD OF IMAGES.                     *
C   **********************************************************************
C
C   WHERE C21,RC21,BETA,WN,PAI,X,H AND F ARE INPUT CONSTANTS
C         FROM MAIN ROUTINE,
C         DISTX IS INPUT VARIABLE FROM MAIN ROUTINE, AND,
C         PAMP AND PHAS ARE OUTPUTS  TO MAIN ROUTINE.
C
C   *  NOTATION
C
C   ----------------------------------------------------------------------
C   SYMBOL          MEANING                                  I/O CHARACTER
C   ----------------------------------------------------------------------
C    PRES   PRESSURE CAUSED BY N TH PAIR OF IMAGES                P,Z
C    P      SUM OF PRES.(IF ALL POSSIBLE PAIR OF IMAGES ARE       P,Z
C              ADDED P IS PAMP
C    ANGLN  ANGLE FORMED BY IMAGE, APEX AND BOTTOM                P,R
C    DD     PARAMETER FOR CONVENIENCE                             P,R
C    R      TOTAL REFLECTION LOSS OF (N') TH IMAGE                P,Z
C    N      N TH IMAGE OR N TH PAIR OF IMAGES                     P,I
C    M      M TH BOTTOM BOUNCE                                    P,I
C    MM     MAXIMUM NUMBERS OF BOTTOM BOUNCES                     P,I
C    THETAO ANGLE  FORMED BY THE N TH IMAGE,THE RECEIVING POINT
C              AND THE BOTTM                                      P,R
C    DR     DISTANCE FROM N TH IMAGE TO THE RECEIVING POINT
C              NORMALIZED BY DIVIDING BY XO                       P,R
C    THETAM GRAZING ANGLE TO THE BOTTOM                           P,R
C              (INTERMEDIATE REFLECTION)
C    CHECK  IDENTIFIES IF GRAZING ANGLE EXCEEDS CRITICAL ANGLE    P,R
C    PSAI   PARAMETER IN REFLECTION COEFFICIENT                   P,Z
C    REFL   REFLECTION COEFFICIENT OF INTERMEDIATE BOUNCE         P,Z
C    REFLNO REFLECTION COEFFICIENT OF THE LAST BOUNCE             P,Z
C    Z      PARAMETER FOR CONVENIENCE                             P,Z
C   ----------------------------------------------------------------------
C
C   **********************************************************************
```

```
C     ((PROGRAM))
C
      SUBROUTINE PRES (DISTX,PAMP,PHAS)
C
C DEFINE CHARACTERS OF VARIABLE,PARAMETERS AND CONSTANTS
C
      IMPLICIT REAL*8(A-H,O-Z),INTEGER(I-N)
      COMPLEX*16 PSAI,REFL,REFLNO,R,Z,PRES,P
      COMMON C21,RC21,BETA,WN,PAI,ANGLO,X,H,F
C
C INITIALIZE COMPLEX PRESSURE P=0.0+J0.0
C
      P=DCMPLX(0.0D0,0.0D0)
C
C RESET COUNTER N=0
C
      N=0
   10 CONTINUE
C
C CALCULATE ANGLN AND DETERMINE IF ANGLE IS IN THE RANGE OR NOT
C  IF ANGLE.GT.PAI GO TO THE NEXT STEP
C
      ANGLN=2.0D0*INT((N+1.)/2.)*BETA+(-1)**N*ANGLO
      IF (ANGLN.GE.PAI) GO TO 50
C
C CALCULATE PARAMETER THETAO AND DR
C
      DD=DCOS(ANGLN)-DISTX
      THETAO=DATAN2(DSIN(ANGLN),DD)
      DR=DSQRT(1.0D0+DISTX**2-2.0D0*DISTX*DCOS(ANGLN))
C
C INITIALIZE THE TOTAL REFLECTION COEFFCIENT R=1.0+J0.0
C
      R=DCMPLX(1.0D0,0.0D0)
C
C DEFINE MM BY EACH PATH WHICH IS DETERMINED BY THE N TH IMAGE
C
      MM=IDINT(ANGLN/(2.0D0*BETA))
      L=MM+1
      DO 40 M=1,L
C
C CALCULATE PARAMETERS THETAM AND CHECK
C
      THETAM=THETAO-2.0D0*DFLOAT(M-1)*BETA
      CHECK=1.0D0-C21**2*DCOS(THETAM)**2
C
C IDENTIFY THE INCIDENT ANGLE WHICH IS LESS THAN THE CRITICAL
C  ANGLE OR NOT
C
      IF(CHECK.GT.0.0D0) GO TO 20
C
C CALCULATE PARAMETER PSAI FOR THE REFLECTION COEFFCIENT OF EACH
C  BOUNCE. THERE ARE TWO WAYS WHICH DEPEND ON THE IDENTIFICATION
C   OF A CHECK
C
      PSAI=DCMPLX(0.0D0,-DSQRT(-CHECK)/DSIN(THETAM))
      GO TO 30
   20 CONTINUE
      PSAI=DCMPLX(DSQRT(CHECK)/DSIN(THETAM),0.0D0)
   30 CONTINUE
```

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```
C CALCULATE REFL
C
      REFL=(RC21-PSAI)/(RC21+PSAI)
C
C IF THE BOUNCE IS THE LAST FROM THE SOURCE,THEN REFLNO=REFL
C
      IF(M.EQ.1) REFLNO=REFL
C
C CALCULATE THE TOTAL REFLECTION COEFFCIENT R BY EACH PATH
C
      R=R*REFL
   40 CONTINUE
C
C CALCULATE PARAMETERS Z AND PRES
C
      Z=DCMPLX(0.0D0,-WN*X*DR)
      PRES=CDEXP(Z)*(-1)**INT((N+1.)/2.)*
     C     (1.0D0+1.0D0/REFLNO)*R/DR
C
C CALCULATE THE PRESSURE P WHICH IS A SUM OF PRES
C
      P=P+PRES
C
C SET TO THE NEXT PAIR OF IMAGE
C
      N=N+1
      GO TO 10
   50 CONTINUE
C
C CALCULATE PA AND PH,AND RETURN BACK TO THE MAIN ROUTINE
C
      PAMP=CDABS(P)
      PREAL=P
      P=P-PREAL
      PP=P*DCMPLX(0.0D0,1.0D0)
      PIMAG=DSIGN(CDABS(P),-PP)
      PHAS=DATAN2(PIMAG,PREAL)
      RETURN
      END
```

FLOW CHART FOR SUBROUTINE 'PRES'
(WEDGE 2)

'PRES'

START

INPUT FROM MAIN

CHARACTE -RIZATION

TRANSFERRED CONSTANTS FROM MAIN

INITIAL CONDITION

CALCULATE φ_n

$\varphi_n < \pi$ NO

YES

CHECK n   EVEN   ODD

CALL REFL(φ_n)

$R(0) = R_0 = REFL(\varphi_0)$
$RA = R(n)$
$= R_{n-2} R_n$

$R = RA$

CALL REFL(φ_n)

$R(1) = R_1 = REFL(\varphi_1)$
$RB = R(n)$
$= R_{n-2} R_n$

$R = RB$

CALCULATE P

CALCULATE $P = \sum P$

SET $n = n + 1$

CALCULATE $|P| \quad P$

OUTPUT FOR MAIN

RETURN TO MAIN

'REFL'

START

$\varphi_n < \theta_c$

$\Psi = \psi_1$

$\Psi = \psi_2$

CALCULATE REFL(φ_n)

RETURN TO SUB.PRES

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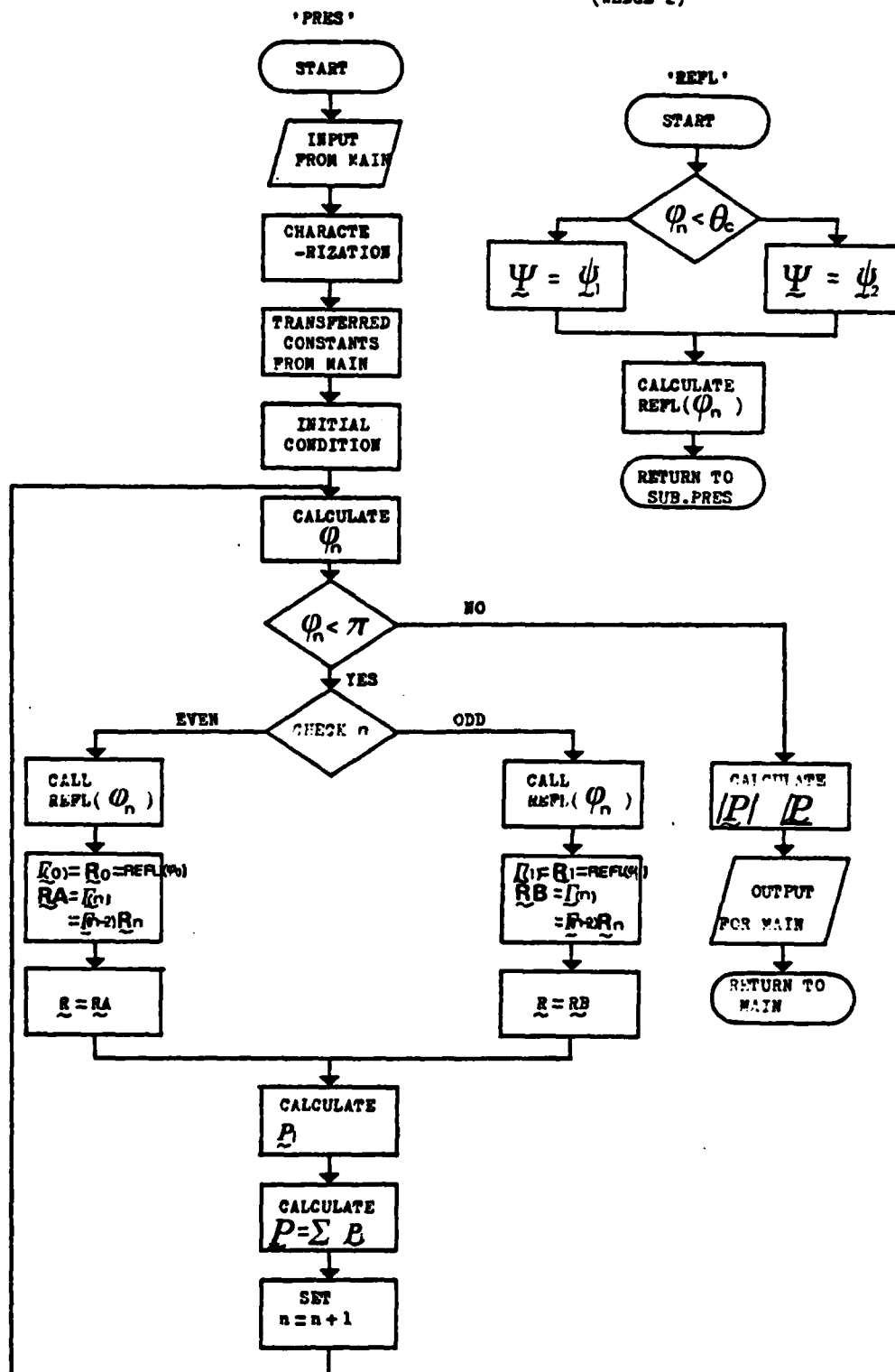
```
C     WEDGE 2
C *** MAIN PROGRAM ***
C
C     **********************************************************************
C     * THIS PROGRAM EMPLOYS THE METHOD OF IMAGES TO OBTAIN THE PRESSURE   *
C     *AND PHASE DISTRIBUTION ALONG THE BOTTOM OF A WEDGE-SHAPED FLUID      *
C     *LAYER,OVERLYING A FAST BOTTOM.                                      *
C     *                                                                    *
C     *  ASSUMPTIONS                                                       *
C     * (1) PLANE WAVE REFLECTION COEFICIENTS.                             *
C     * (2) SOURCE IS A POINT SOURCE.                                      *
C     * (3) SOURCE IS A DISTANT SOURCE.                                    *
C     **********************************************************************
C
C     *  NOTATION
C        I/O   OUT=OUTPUT
C              IN =INPUT
C        CHARACTER
C              V  =VARIABLE
C              P  =PARAMETER
C              C  =CONSTANT
C
C              R  =REAL NUMBER
C              I  =INTEGER
C              Z  =COMPLEX NUMBER
C
C        ----------------------------------------------------------------
C        SYMBOL        MEANING                               I/O CHARACTER
C        ----------------------------------------------------------------
C        PA     PRESSURE AMPLITUDE ON THE BOTTOM             OUT   V,R
C        PH     PRESSURE PHASE ON THE BOTTOM                 OUT   V,R
C        C1     SOUND SPEED IN MEDIUM 1 (WEDGE)              IN    C,R
C        C2     SOUND SPEED IN MEDIUM 2 (BOTTOM)             IN    C,R
C        RO1    DENSITY OF MEDIUM 1                          IN    C,R
C        RO2    DENSITY OF MEDIUM 2                          IN    C,R
C        BETA   SLOPE ANGLE OF BOTTOM                        IN    C,R
C        XX     DISTANCE FROM THE APEX IN METER              OUT   V,R
C                 AT WHICH PA AND PH ARE CALCULATED
C        F      DRIVING FREQUENCY                            IN    C,R
C        ANGLC  CRITICAL ANGLE                                     C,R
C        HO     LOWEST POSSIBLE MODE CUT OFF DEPTH IN METERS        C,R
C        XO     LOWEST POSSIBLE MODE CUT OFF DISTANCE FROM THE      C,R
C                 APEX IN METERS
C        X1     DISTANCE FROM APEX AT WHICH CALCULATION STARTS IN   C,R
C        X2     DISTANCE FROM APEX AT WHICH CALCULATION STOPS  IN   C,R
C        DX     RANGE OF CALCULATION IN METERS                     C,R
C        N      NUMBER OF POINTS FOR WHICH PA AND PH ARE     IN    C,I
C                 CALCULATED
C        PAI    3.1415.......                                      C,R
C        WN     WAVE NUMBER                                        C,R
C        C21    SOUND SPEED RATIO =C2/C1                           C,R
C        RC21   ACOUSTIC IMPEDANCE RATIO                           C,R
C        ANGLO  ANGLE FORMED BY SOURCE, APEX AND BOTTOM      IN    C,R
C        DISTX  TEMPORARY FOR XX(I)/X                              P,R
C        PAMP   TEMPORARY FOR PA                                   P,R
C        PHAS   TEMPORARY FOR PH                                   P,R
C        ----------------------------------------------------------------
C
C
C     **********************************************************************
```

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```
C      ((PROGRAM))
C
C DEFINE CHARACTERS OF VARIABLES,PARAMETERS AND CONSTANTS
C
       IMPLICIT REAL*8(A-H,O-Z),INTEGER(I-N)
       DIMENSION PA(101),PH(101),XX(101)
       COMMON C21,RC21,BETA,WN,PAI,ANGLO,F
C
C READ IN DATA
C
       READ(5,100) C1,C2,RO1,RO2,BETA
       READ(5,500) X1,X2,F,ANGLO
       READ(5,600) N
C
C CALCULATE AND WRITE CONSTANTS
C
       WRITE (6,700) C1,C2,RO1,RO2,X1,X2,BETA,F,ANGLO
       ANGLC=DARCOS(C1/C2)
       HO=C1/(4.0DO*F*DSIN(ANGLC))
       XO=HO/DSIN(BETA)
       WRITE(6,400) HO,XO
C
C CALCULATE RANGE DX
C
       DX=X2-X1
C
C CALCULATE CONSTANTS FOR SUBROUTINE
C
       PAI=4.0DO*DATAN(1.0DO)
       WN=2.0DO*PAI*F/C1
       C21=C2/C1
       RC21=C2*RO2/(C1*RO1)
       WRITE (6,200)
C
C CALCULATE PRESSURES WITH RESPECT TO DISTANCE XX
C  WHERE XX IS CHANGED BY INCRIMENT
C
       L=N+1
       DO 10 I=1,L
       DISTX=X1+DFLOAT(I-1)*DX/DFLOAT(N)
       XX(I)=DISTX
       CALL PRES (DISTX,PAMP,PHAS)
       PA(I)=PAMP
       PH(I)=PHAS
       WRITE (6,300) XX(I),PA(I),PH(I)
   10 CONTINUE
C
C PLOT THE PRESSURE AMPLITUDE VS. DISTANCE XX
C  AND PRESSURE PHASE VS. DISTANCE XX
C
       WRITE (6,800)
       WRITE (6,700) C1,C2,RO1,RO2,X1,X2,BETA,F,ANGLO
       CALL DPLTP (XX,PA,N,0)
       WRITE (6,800)
       WRITE (6,700) C1,C2,RO1,RO2,X1,X2,BETA,F,ANGLO
       CALL DPLTP (XX,PH,N,0)
```

```
C I/O FORMATS
C
  100 FORMAT (5F15.0)
  200 FORMAT (//22X,'DISTANCE',17X,'PRESSURE AMP.',12X,'PHASE ANGLE'//)
  300 FORMAT (   20X,D15.7,10X,D15.7,10X,D15.7)
  400 FORMAT (//20X,'LOWEST POSSIBLE MODE CUT OFF DEPTH',9X,D15.7
     C            /20X,'DISTANCE FROM APEX       ',20X,D15.7//)
  500 FORMAT (4F15.0)
  600 FORMAT (I3)
  700 FORMAT (//15X,'INPUT DATA',25X,'WEDGE2'//
     C            25X,'C1  =',D15.7,5X,'C2  =',D15.7/
     C            25X,'RO1 =',D15.7,5X,'RO2 =',D15.7/
     C            25X,'X1  =',D15.7,5X,'X2  =',D15.7/
     C            25X,'BETA=',D15.7,5X,'F   =',D15.7/
     C            42X,'SOURCE ANGLE=',D15.7/)
  800 FORMAT (1H1)
      STOP
      END
```

30

```
C *** SUB PROGRAM ***
C
C  ***********************************************************************
C  * SUBROUTINE 'PRES' CALCULATES THE PRESSURE AMPLITUDE AND PHASE AT A*
C  *POINT ON THE BOTTOM BY USING METHOD OF IMAGES.                     *
C  ***********************************************************************
C
C  WHERE C21,RC21,BETA,WN,PAI,ANGLO AND F ARE INPUT CONSTANTS
C        FROM MAIN ROUTINE,
C        DISTX IS INPUT VARIABLE FROM MAIN ROUTINE, AND,
C        PAMP AND PHAS ARE OUTPUTS  TO MAIN ROUTINE.
C
C  *  NOTATION
C
C     -----------------------------------------------------------------
C     SYMBOL        MEANING                                 I/O CHARACTER
C     -----------------------------------------------------------------
C     PRES    PRESSURE CAUSED BY N TH PAIR OF IMAGES              P,Z
C     P       SUM OF PRES.(IF ALL POSSIBLE PAIR OF IMAGES ARE     P,Z
C               ADDED P IS PAMP
C     ANGLN   ANGLE FORMED BY IMAGE, APEX AND BOTTOM              P,R
C     N       N TH IMAGE OR N TH PAIR OF IMAGES                   P,I
C     IFLAG   IFLAG=0 WHEN N IS EVEN
C             IFLAG=1 WHEN N IS ODD                               P,I
C     REFL    REFLECTION COEFFICIENT OF INTERMEDIATE BOUNCE       P,Z
C             FUNCTION PROGRAM REFL (ANGLN)
C     CHECK   PARAMETER FOR FUNCTION REFL                         P,R
C             (IDENTIFY IF GRAZING ANGLE EXCEEDS CRITICAL ANGLE)
C     PSAI    PARAMETER FOR FUNCTION REFL                         P,Z
C     REFLNO  REFLECTION COEFFICIENT OF THE LAST BOUNCE           P,Z
C     R       TOTAL REFLECTION LOSS OF (N') TH IMAGE              P,Z
C     RA      TEMPORARY FOR R WHEN N IS EVEN                      P,Z
C     RB      TEMPORARY FOR R WHEN N IS ODD                       P,Z
C     Z       PARAMETER FOR CONVENIENCE                           P,Z
C     -----------------------------------------------------------------
C
C  ***********************************************************************
```

```
C
C      ((PROGRAM))
C
       SUBROUTINE PRES (DISTX,PAMP,PHAS)
C
C DEFINE CHARACTERS OF VARIABLE,PARAMETERS AND CONSTANTS
C
       IMPLICIT REAL*8(A-H,O-Z),INTEGER(I-N)
       COMPLEX*16 R,RA,RB,REFL,REFLNO,P,PRES,Z
       COMMON C21,RC21,BETA,WN,PAI,ANGLO,F
C
C INITIALIZE COMPLEX PRESSURE P=0.0+J0.0
C
       P=DCMPLX(0.0D0,0.0D0)
C
C RESET COUNTER N=0 , RESET IFLAG=0
C
       N=0
       IFLAG=0
   10 CONTINUE
C
C CALCULATE ANGLN AND DETERMINE IF ANGLE IS IN THE RANGE OR NOT
C  IF ANGLE.GT.PAI GO TO THE NEXT STEP
C
       ANGLN=2.0D0*INT((N+1.)/2.)*BETA+(-1)**N*ANGLO
       IF (ANGLN.GE.PAI) GO TO 80
C
C CALCULATE THE TOTAL REFLECTION COEFFICIENT R BY EACH PATH
C
       IF (IFLAG.EQ.1) GO TO 40
       REFLNO=REFL(ANGLN)
       IF (N.GE.2) GO TO 20
       RA=REFLNO
       R=RA
       GO TO 30
   20 CONTINUE
       RA=RA*REFLNO
       R=RA
   30 CONTINUE
       IFLAG=1
       GO TO 70
   40 CONTINUE
       REFLNO=REFL(ANGLN)
       IF (N.GE.2) GO TO 50
       RB=REFLNO
       R=RB
       GO TO 60
   50 CONTINUE
       RB=RB*REFLNO
       R=RB
   60 CONTINUE
       IFLAG=0
   70 CONTINUE
```

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```
C CALCULATE PARAMETERS Z AND PRES
C
      Z=DCMPLX(0.0D0,WN*DISTX*DCOS(ANGLN))
      PRES=CDEXP(Z)*(-1)**INT((N+1.)/2.)*
     C     (1.0D0+1.0D0/REFLNO)*R
C
C CALCULATE THE PRESSURE P WHICH IS A SUM OF PRES
C
      P=P+PRES
C
C SET TO THE NEXT PAIR OF IMAGE
C
      N=N+1
      GO TO 10
   80 CONTINUE
C
C CALCULATE PA AND PH,AND RETURN BACK TO THE MAIN ROUTINE
C
      PAMP=CDABS(P)
      PREAL=P
      P=P-PREAL
      PP=P*DCMPLX(0.0D0,1.0D0)
      PIMAG=DSIGN(CDABS(P),-PP)
      PHAS=DATAN2(PIMAG,PREAL)
      RETURN
      END
```

```
C *** SUB PROGRAM ***
C
C     ((PROGRAM))
C
      FUNCTION REFL (ANGLN)
C
C DEFINE CHARACTERS OF VARIABLES, PARAMETERS AND CONSTANTS
C
      IMPLICIT REAL*8(A-H,O-Z)
      COMPLEX*16 REFL,PSAI
      COMMON C21,RC21
C
C CALCULATES PARAMETER CHECK
C
      CHECK=1.0D0-C21**2*DCOS(ANGLN)**2
C
C IDENTIFY THE GRAZING ANGLE WHICH IS LESS THAN THE CRITICAL
C  ANGLE OR NOT
C
      IF (CHECK.GT.0.0D0) GO TO 10
C
C CALCULATE PARAMETER PSAI FOR THE REFLECTION COEFFCIENT OF EACH
C  BOUNCE. THERE ARE TWO WAYS WHICH DEPEND ON THE IDENTIFICATION
C   OF A CHECK
C
      PSAI= DCMPLX(0.0D0,-DSQRT(-CHECK)/DSIN(ANGLN))
      GO TO 20
   10 CONTINUE
      PSAI= DCMPLX(DSQRT(CHECK)/DSIN(ANGLN),0.0D0)
   20 CONTINUE
C
C CALCULATE REFL
C
      REFL=(RC21-PSAI)/(RC21+PSAI)
C
C RETURN BACK TO THE SUBROUTINE PRES
C
      RETURN
      END
```

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